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zation is known, and turning the analyzing prism until the tints become equal. The beam whose plane is required is then introduced, and when the equality of tints has been re-established, the angle through which the prism has revolved, read off on a graduated circle, gives the inclination of the required plane to the known plane. This mode of determining the zero, a process which for perfect accuracy ought to be repeated with each new set of observations, possesses the advantage of eliminating the personal equation of the observer. In examining a beam of any considerable magnitude, there will be found in different persons a tendency to think one part of the image darker than another, even when there is no real difference. With different observers, and even in the same person at different times, the part of the image thus preferred may be different, and if the zero were determined once for all, this might occasion sensible error. But as, in the method here given, such a preference will equally affect the position of the zero, it can have no influence on the final result.

In the first prism which the author procured to be constructed, the angle between the planes CP, CP′ was about 7°. With this prism the range of error in the determination of a plane of polarization was 7′, the light employed being the diffused light of the sky. Although this was a very much smaller range than the author had ever been able to attain with a Nicol's prism, it seemed that a greater amount of accuracy might be obtained, and as the brightness of the image appeared to be too great, a prism was constructed in which this angle was but half of its former value. With this prism, and with the same kind of light, the author finds that he can determine the position of the plane of polarization to 1′.

With direct solar light, and a prism in which the planes are still closer, a greater degree of accuracy may certainly be obtained; in fact, it can be shown that by diminishing this angle and increasing the brightness of the light, so as to preserve unchanged the intensity of the image, the sensibility of the prism will vary as $\cot \frac{\theta}{2}$, θ being the angle in question.

The author has given direction for the construction of an instrument in which this angle shall be one-half of that last mentioned, but as the polarimeter at his disposal is not made to measure an angle less than 1′, a greater degree of accuracy would not be of any practical advantage.

The prisms here described were executed by Mr. J. Bryson, of Prince's-street, Edinburgh, in a manner which left nothing to be desired.

Professor Kinahan read a paper, entitled, "Descriptions of Crangonidæ and Galatheidæ inhabiting the Irish seas, with Observations on the general homologies of the groups, and the propriety of subdividing the Britannic species of the former."

Sir William Rowan Hamilton, LL. D., M. R. I. A., communicated *the following portion of his paper*—

13. Proceeding to *space*, let A, B, C, D be the four corners of a given triangular pyramid, and let E be any fifth given point, which is not on any one of the four faces of that pyramid. Let P be any sixth point of space; and let $xyzw$ be four positive or negative numbers, such that

$$\frac{x}{w} = (\mathrm{BC.\ AEDP}),\ \frac{y}{w} = (\mathrm{CA.\ BEDP}),\ \frac{z}{w} = (\mathrm{AB.\ CEDP});$$

the right-hand member of these equations representing *anharmonics of pencils of planes*, in a way which is easily understood, with the help of the definition (5) of the symbol (ABCD). Then I call x, y, z, w (or any numbers proportional to them), the *Anharmonic Co-ordinates* of the *Point* P, with respect to what may be said to be the *Unit-Pyramid*, ABCD, because its corners may (on the present plan) be thus denoted,

$$\mathrm{A} = (1,\ 0,\ 0,\ 0);\ \mathrm{B} = (0,\ 1,\ 0,\ 0);\ \mathrm{C} = (0,\ 0,\ 1,\ 0);\ \mathrm{D} = (0,\ 0,\ 0,\ 1);$$

and with respect to that fifth given point E, which may be called the *Unit-Point*, because its symbol, in the present system, may be thus written:—

$$\mathrm{E} = (1,\ 1,\ 1,\ 1).$$

And I denote the general or variable point by the symbol,

$$\mathrm{P} = (x,\ y,\ z,\ w).$$

14. When we have thus five given points, A . . . E, of which no four are situated in any common plane, we can connect any two of them by a right line, and the three others by a plane, and determine the point in which these last intersect each other, deriving in this way a system of ten lines, ten planes, and ten points, whereof the latter may be thus denoted:—

$$\mathrm{A}' = \mathrm{BC}\cdot\ \mathrm{ADE} = (0,\ 1,\ 1,\ 0),\ \ \mathrm{B}' = \&\mathrm{c}.,\ \mathrm{C}' = \&\mathrm{c}.;$$
$$\mathrm{A}_1 = \mathrm{AE}\cdot\ \mathrm{BCD} = (0,\ 1,\ 1,\ 1),\ \ \mathrm{B}_1 = \&\mathrm{c}.,\ \mathrm{C}_1 = \&\mathrm{c}.;$$
$$\mathrm{A}_2 = \mathrm{AD}\cdot\ \mathrm{BCE} = (1,\ 0,\ 0,\ 1),\ \ \mathrm{B}_2 = \&\mathrm{c}.,\ \mathrm{C}_2 = \&\mathrm{c}.;$$
$$\mathrm{D}_1 = \mathrm{DE}\cdot\ \mathrm{ABC} = (1,\ 1,\ 1,\ 0),;$$

and the harmonic conjugates of these last points, with respect to the ten given lines on which they are situated, may on the same plan be represented by the following symbols:—

$$\mathrm{A}'' = (0,\ 1,\ -1,\ 0),\ \ \mathrm{B}'' = \&\mathrm{c}.\ \mathrm{C}'' = \&\mathrm{c}.$$
$$\mathrm{A}'_1 = (2,\ 1,\ \ \ 1,\ 1),\ \ \mathrm{B}'_1 = \&\mathrm{c}.\ \mathrm{C}'_1 = \&\mathrm{c}.$$
$$\mathrm{A}'_2 = (1,\ 0,\ 0,\ -1),\ \ \mathrm{B}'_2 = \&\mathrm{c}.\ \mathrm{C}'_2 = \&\mathrm{c}.$$
$$\mathrm{D}'_1 = (1,\ 1,\ 1,\ \ \ 2);$$

so that

$$(\mathrm{BA'CA''}) = \ldots = (\mathrm{EA_1AA'_1}) = \ldots = (\mathrm{DA_2AA'_2}) = \ldots = (\mathrm{ED_1DD'_1}) = -1.$$

15. Let any plane Π intersect the three given lines DA, DB, DC in points Q, R, S; and let *lmnr* be any positive or negative numbers, such that

$$\frac{l}{r} = (\mathrm{DA'_2AQ}), \frac{m}{r} = (\mathrm{DB'_2BR}), \frac{n}{r} = (\mathrm{DC'_2CS});$$

then I call l, m, n, r, or any numbers proportional to them, the *Anharmonic Coordinates* of the *Plane* Π; which *plane* I also denote by the *Symbol*,

$$\Pi = [l, m, n, r].$$

In particular the four faces of the unit pyramid come thus to be denoted by the symbols,

$$\mathrm{BCD} = [1, 0, 0, 0], \mathrm{CAD} = [0, 1, 0, 0], \mathrm{ABD} = [0, 0, 1, 0], \mathrm{ABC} = [0, 0, 0\ 1];$$

and the six planes through its edges, and through the unit point, are denoted thus :—

$$\mathrm{BCE} = [1, 0, 0, -1]; \mathrm{CAE} = [0, 1, 0, -1]; \mathrm{ABE} = [0, 0, 1, -1];$$
$$\mathrm{ADE} = [0, 1, -1, 0]; \mathrm{BDE} = [-1, 0, 1, 0]; \mathrm{CDE} = [1, -1, 0, 0];$$

in connexion with which last planes it may be remarked that we have, generally, as a consequence of the foregoing definitions, the formulæ,

$$\frac{n}{m} = (\mathrm{BA''CL}), \frac{l}{n} = (\mathrm{CB''AM}), \frac{m}{l} = (\mathrm{AC''BN}),$$

if L, M, N be the points in which a variable plane Π intersects the sides BC, &c., of the given triangle ABC: as we have also, generally,

$$\frac{z}{y} = (\mathrm{AD}, \mathrm{CEBP}), \frac{x}{z} = (\mathrm{BD}.\ \mathrm{AECP}), = \frac{y}{x}\ (\mathrm{CD}.\mathrm{BEAP}).$$

16. If a *point*, P = $(xyzw)$, be situated *on a plane*, Π = $[lmnr]$, then I find that the following relation between their co-ordinates exists, which is entirely analogous to that already assigned (6) for the case of a *point* and *line* in a given plane, and is of fundamental importance in the application of the present *Anharmonic Method* to *space:*

$$lx + my + nz + rw = 0;$$

or in words, "*the sum of the products of corresponding co-ordinates*, of point and plane, *is zero*."

For example, all planes through the unit point (1, 1, 1, 1) are subject to the condition,

$$l + m + n + r = 0,$$

as may be seen for the six planes (15) already drawn through that point E; and the six points $\mathrm{A''}$ $\mathrm{B''}$ $\mathrm{C''}$ $\mathrm{A'_2}$ $\mathrm{B'_2}$ $\mathrm{C'_2}$ (14), in which the six edges BC, CA, AB, DA, DB, DC, of the given or unit pyramid ABCD, intersect the six corresponding edges of the inscribed and homologous pyramid $\mathrm{A_1B_1C_1D_1}$, with the unit point E for their centre of homology, are all ranged on one common plane of homology, of which the *equation* and the *symbol* may be thus written,

$$x + y + r + w = 0, \ [\mathrm{E}] = [1, 1, 1, 1],$$

and which may be called (comp. 4) the *Unit-Plane.*

17. Any four collinear points, P_0, P_1, P_2, P_3, have their anharmonic symbols connected by two equations of the forms,

$$(\mathrm{P}_1) = t\,(\mathrm{P}_0) + u\,(\mathrm{P}_2), \ \mathrm{P}_3 = t'\,(\mathrm{P}_0) + u'\,(\mathrm{P}_2)$$

each including four ordinary linear equations between the co-ordinates of the four points, such as

$$x_1 = tx_0 + ux_2, \ y_1 = ty_0 + uy_2, \ \&\text{c.};$$

and the anharmonic of their *group* is then found to be expressed by the formula,

$$(\mathrm{P}_0, \mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3) = \frac{ut'}{tu'}.$$

And similarly, if any four planes $\Pi_0 \,..\, \Pi_3$ be collinear (that is, if they have any one right line common to them all), their symbols satisfy two linear equations of the corresponding forms,

$$[\Pi_1] = t\,[\Pi_0] + u\,[\Pi_2], \ [\Pi_3] = t'\,[\Pi_0] = u'\,[\Pi_2];$$

and the anharmonic of the *pencil* is,

$$(\Pi_0\Pi_1\Pi_2\Pi_3) = \frac{ut'}{tu'}.$$

18. If $\phi(xyzw)$ be any *rational fraction,* the numerator and denominator of which are any two given homogeneous and linear functions of the co-ordinates of a *variable point;* and if we determine a *line* Λ, and *three planes* Π_0, Π_1, Π_2 through that line, by the four *local* equations,

$$\phi = \frac{0}{0}, \ \phi = \infty, \ \phi = 1, \ \phi = 0;$$

then I find that the function ϕ may be expressed as the anharmonic of a *pencil of planes,* as follows:—

$$\phi(xyzw) = (\Pi_0\Pi_1\Pi_2\Pi);$$

where Π is the variable plane $\Lambda\mathrm{P}$, which passes through the fixed line Λ, and through the variable point $\mathrm{P} = (xyzw)$.

19. And in like manner, as the *geometrical dual* (9) of this last theorem (18), if $\Phi(lmnr)$ be any rational fraction, of which the numerator and denominator are any two given functions, homogeneous and linear, of the co-ordinates of a *variable plane;* and if we determine a *line* Λ, and *three points* P_0, P_1, P_2 on that line, by the four *tangential* equations,

$$\Phi = \frac{0}{0}, \ \Phi = \infty, \ \Phi = 1, \ \Phi = 0;$$

I find that the proposed function Φ may then be thus expressed as the anharmonic of a *group of points.*

$$\Phi(lmnr) = (\mathrm{P}_0\mathrm{P}_1\mathrm{P}_2\mathrm{P});$$

P here denoting the variable point $\Lambda \cdot \Pi$, in which the fixed line Λ intersects the variable plane $\Pi = [lmnr]$.

20. All problems respecting *intersections of lines with planes*, &c., are resolved, with the help of the Fundamental Theorem (16) respecting the relation which exists between the anharmonic co-ordinates of point and plane, as easily by the present method, as by the known method of *quadriplanar* co-ordinates (10); and indeed, by the very *same mechanism*, of which it is therefore unnecessary here to speak.

But it may be proper to say a few words respecting the application of the anharmonic method to *Surfaces* (7); although here again the known mechanism of *calculation* may in great part be preserved unchanged, and only the *interpretations* need be *new*.

21. In general, it is easy to see (comp. 7) that, in the present method, as in older ones, the *order* of a curved surface is denoted by the *degree* of its *local equation*, $f(xyzw) = 0$; and that the *class* of the same surface is expressed, in like manner, by the degree of its *tangential equation*, $F(lmnr) = 0$: because the *former* degree (or dimension) determines the *number of points* (distinct or coincident, and real or imaginary), in which the surface, considered as a locus, is *intersected* by an arbitrary right *line;* while the *latter* degree determines the *number of planes* which can be drawn *through* an arbitrary right line, so as to touch the same surface, considered as an *envelope*. It may be added, that I find the *partial derivatives of each* of these two functions, f and F, to be proportional to the *co-ordinates* which enter as variables into the *other;* thus we may write

$$[D_x f,\ D_y f,\ D_z f,\ D_w f],$$

as the symbol (15) of the *tangent plane* to the *locus* f, at the point $(xyzw)$; and

$$(D_l F,\ D_m F,\ D_n F,\ D_r F),$$

as a symbol for the *point of contact* of the *envelope* F, with the plane $[lmnr]$: whence it is easy to conceive how problems respecting the *polar reciprocals* of *surfaces* are to be treated.

22. As a very simple *example*, the surface of the *second order* which passes through the *nine points*, above called ABCDEA$'$A$_2$C$'$C$_2$, is easily found to have for its *local* equation, $0 = f = xz - yw$; whence the co-ordinates of its tangent plane are, $l = D_x f = z$, $m = D_y f = -w$, $n = D_z f = x$, $r = D_w f = -y$, and its *tangential* equation is, therefore, $0 = F = ln - mr$, so that it is also a surface of the *second class*. In fact it is the *hyperboloid* on which the gauche quadrilateral ABCD is superscribed, and which passes also through the point E; and the known *double generation*, and *anharmonic properties*, of this surface, may easily be deduced from either of the foregoing forms of its anharmonic equation, whereof the first may (by 13, 15) be expressed as an equality between the anharmonic functions of two *pencils of planes*, in either of the two following ways:— (BC. AEDP) = (DA. BECP); (AB. CEDP) = (CD. BEAP).

ERRATUM.—In line 5 of Art. 12, for *a* read *u*.